# What is the Electron Spin?

**By Gengyun Li**

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# What is the electron spin?

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# 1

# Introduction

The electron has both an intrinsic electric field and an intrinsic magnetic field. The electron's intrinsic electromagnetic field has both energy and angular momentum. In our book, we have the assumption that the electron has the electromagnetic origin, the electron spin is the electron's electromagnetic field angular momentum, and the electron's self-energy is the electron's electromagnetic field energy.

The simplest model of electron spin is a spinning electrically charged ball, similar to the earth rotating about its own axis. In this model, the electron's rest energy equals the electrostatic potential energy of a sphere of charge *e* with radius $r_0$:

$$m_e c^2 = \frac{e^2}{4\pi\varepsilon_0 r_0} \tag{1.1}$$

In which $r_0$ is the classical radius of the electron, $m_e$ is the electron's mass.

Combine electron mass $m_e$ with radius $r_0$, and we get electron spin angular momentum:

$$m_e v r_0 = \frac{\hbar}{2} \tag{1.2}$$

In which v is the classical velocity for electron spin.

Then we can obtain the electron's classical velocity:

$$v = \frac{\hbar}{m_e r_0} \tag{1.3}$$

Combine this with equation (1.1), thus:

$$v = \frac{4\pi\varepsilon_0 c^2 \hbar}{e^2} \tag{1.4}$$

And so:

$$v = \frac{c}{\alpha} \tag{1.5}$$

In which $\alpha$ is fine structure constant:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{1.6}$$

As we know:

$$\alpha \approx \frac{1}{137} \tag{1.7}$$

Therefore:

$$\frac{v}{c} \approx 137 \tag{1.8}$$

Based on the above model of electron spin, the classical electron velocity is much greater than the velocity of light! Why did the spinning electric charge ball model fail for the electron spin? The basic assumption of the above electron spin model is that the electron's mass originates from electric field energy.

In our book, we have an assumption that the electron has electromagnetic origin. The electron's self-energy is the electron's electromagnetic field energy, which mainly comes from electron magnetic field energy, in comparison to the electron's magnetic field energy, the electron's electric field energy is much smaller. This is one of the major reasons why the above electron spin model failed.

We no longer regard the electron as a point-like particle. Instead, we assume that electrons have internal structure; inside the electron it has continuum spherical distribution of both electric charge and magnetic charge. Based upon the charge distribution, and also according to the Gauss Laws for electric field and magnetic field, we calculate the electric field and magnetic field distribution inside the electron, and then we calculate electromagnetic field energy and angular momentum of the electron.

Then we make the assumption that both the electron mass and spin have electromagnetic origin. Thus we obtain the electron's self-energy from the electromagnetic field energy, and the electron spin from the electron's electromagnetic field angular momentum.

Then we extend our electron's electromagnetic model to the proton and neutron, the proton and neutron also have electromagnetic origin; and we continue our electromagnetic model to the hydrogen and helium atoms in our book.

In our book, we also provide a possible solution for the hydrogen atom spectrum, in which the hydrogen atom spectrum is regarded as the hydrogen electromagnetic field stationary wave energy spectrum.

The particles that comprise all physical materials include electrons, protons and neutrons; all of which have electromagnetic origin. Therefore, all materials have electromagnetic origin.

Chapter

# 2 The electric charge distribution inside of the electron.

We no longer regard the electron as a point-like particle. The electron has an internal structure, and the electric charge inside the electron has continuum distribution.

Here is one of our most basic assumptions about the electron; the electric charge distribution inside of the electron has the following equation:

$$\rho_e(r,\theta) = -\frac{e}{\pi^2 a_e^{\ 2}}\frac{1}{r}\exp(-\frac{r}{a_e})\sin\theta \qquad (2.1)$$

We use the spherical polar coordinates in our book, in which r is the radial coordinate, $\theta$ is the polar angle, e is the electric charge of the electron, $a_e$ is the electron radius constant.

By integrating the electric charge density equation (2.1), we can find the volume electric charge:

$$Q_e(r,\theta) = \int_V \rho_e(r,\theta) dr^3 \qquad (2.2)$$

In which $dr^3 = 2\pi r^2 \sin\theta d\theta dr$, thus:

$$Q_e(r,\theta) = -\int_V \frac{e}{\pi^2 a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e})\sin\theta 2\pi r^2 \sin\theta d\theta dr \qquad (2.3)$$

$$Q_e(r,\theta) = -\frac{2e}{\pi}\int_V \frac{r}{a_e}\exp(-\frac{r}{a_e})d(\frac{r}{a_e})\sin^2\theta d\theta \qquad (2.4)$$

$$Q_e(r,\theta) = -\frac{2e}{\pi}[\int_0^r \frac{r}{a_e}\exp(-\frac{r}{a_e})d(\frac{r}{a_e})][\int_0^\theta \sin^2\theta d\theta] \qquad (2.5)$$

$$Q_e(r,\theta) = -e[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})]\frac{1}{\pi}(\theta - \frac{1}{2}\sin 2\theta) \qquad (2.6)$$

The equation (2.6) is the electric charge space distribution equation.

From equation (2.6), we find out that:

When $\theta = \pi$, therefore:

$$Q_e(r,\pi) = -e[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{2.7}$$

As we can see, the electric charge distribution is a kind of cumulative gamma distribution [B]

Equation (2.7) is the electric charge equation within the sphere of radius r.
When $r \to \infty$, thus:

$$Q_e = -e \tag{2.8}$$

Thus we can see that the electron as a whole has one unit negative electric charge e.

Chapter

# 3 The magnetic charge distribution inside of the electron.

In our electron model, we no longer regard the electron as a point-like particle. Similar to the electron's electric charge, which has continuum distribution inside of the electron, we make one of our basic assumptions here that the magnetic charge has continuum distribution inside of the electron. The distribution equation is as follows:

$$\rho_m(r,\theta) = -\frac{g}{\pi a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e})\cos\theta \tag{3.1}$$

By integrating the magnetic charge density equation (3.1), we can get the volume magnetic charge:

$$Q_m(r,\theta) = \int_V \rho_m(r,\theta)dr^3 \tag{3.2}$$

In which $dr^3 = 2\pi r^2 \sin\theta d\theta dr$ thus

$$Q_m(r,\theta) = -\int_V \frac{g}{\pi a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e})\cos\theta 2\pi r^2 \sin\theta d\theta dr \tag{3.3}$$

Therefore,

$$Q_m(r,\theta) = -2g\int_V \frac{r}{a_e}\exp(-\frac{r}{a_e})d(\frac{r}{a_e})\cos\theta\sin\theta d\theta \tag{3.4}$$

Thus:

$$Q_m(r,\theta) = -2g[\int_0^r \frac{r}{a_e}\exp(-\frac{r}{a_e})d(\frac{r}{a_e})][\int_0^\theta \cos\theta\sin\theta d\theta] \tag{3.5}$$

Consequently,

$$Q_m(r,\theta) = -g[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})]\frac{1}{2}(1-\cos 2\theta) \tag{3.6}$$

Equation (3.6) is the electron magnetic charge space distribution equation. From the above equation (3.6), the radial part of the distribution is a kind of cumulative gamma distribution function *[B].

From equation (3.6), we find out that:
When $\theta = \pi$ , Thus:

$$Q_m(r,\pi) = 0 \tag{3.7}$$

From equation (3.7), we learn that within the sphere of any radius r, the magnetic charge total is always zero.

We find out that inside of the electron, within the sphere of any radius r, the magnetic charge as a whole is always zero, even though magnetic charge distribution exists inside of the electron.

When $\theta = \frac{\pi}{2}$, thus:

$$Q_m(r,\frac{\pi}{2}) = -g[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{3.8}$$

The above equation (3.8) is the electron north pole area magnetic charge distribution within the sphere of radius r, from which we can see the magnetic charge distribution is also the cumulative gamma distribution [B].

When $r \to \infty$ thus:

$$Q_m = -g \tag{3.9}$$

Which shows us that the electron has one unit negative magnetic charge ‘g’ in the whole north pole area ($0 \le \theta < \frac{\pi}{2}$).
Inside the electron, the whole area of the north pole, has one unit negative magnetic charge ‘g’.

As we know, the electron has total zero magnetic charge, so the electron whole south pole area ($\frac{\pi}{2} \le \theta < \pi$) has one unit positive magnetic charge ‘g’.

Inside the electron, the whole area of south pole has one unit positive magnetic charge ‘g’.

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Chapter

# 4 The electric field inside of the electron

As we know, the Gauss Law of Electric Field defines the relationship between electric charge and electric field as follows:

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{4.1}$$

In which $\vec{E}$ is the electric field strength, and $\rho_e$ is the electric charge density.
If we combine the Gauss Law of Electric Field equation (4.1) and the electric charge density distribution equation (2.1), we will obtain the follows electric field solution:

$$\vec{E} = \frac{e}{\pi^2 \varepsilon_0 a_e} \frac{1}{r} \exp(-\frac{r}{a_e})(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{4.2}$$

Below, we will prove that the above equation (4.2) satisfies the Gauss Law of Electric Field.

As we know the gradient $\nabla$ in spherical coordinate is:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial\phi}$$

From equation (4.2), thus:

$$\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0 \pi^2 a_e}\{\frac{\partial}{\partial r}[\frac{1}{r}\exp(-\frac{r}{a_e})]\sin\theta - \frac{1}{r^2}\exp(-\frac{r}{a_e})\frac{\partial}{\partial\theta}\cos\theta\} \tag{4.3}$$

Thus:

$$\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0 \pi^2 a_e}\{\frac{\partial}{\partial r}[\frac{1}{r}\exp(-\frac{r}{a_e})]\sin\theta + \frac{1}{r^2}\exp(-\frac{r}{a_e})\sin\theta\} \tag{4.4}$$

$$\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0 \pi^2 a_e}\{[-\frac{1}{r^2}\exp(-\frac{r}{a_e}) - \frac{1}{ra_e}\exp(-\frac{r}{a_e})]\sin\theta + \frac{1}{r^2}\exp(-\frac{r}{a_e})\sin\theta\} \tag{4.5}$$

Therefore:

$$\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0 \pi^2 a_e} \{[-\frac{1}{r^2} \exp(-\frac{r}{a_e}) - \frac{1}{r a_e} \exp(-\frac{r}{a_e})] \sin\theta + \frac{1}{r^2} \exp(-\frac{r}{a_e}) \sin\theta\} \qquad (4.6)$$

And so:

$$\nabla \cdot \vec{E} = -\frac{e}{\varepsilon_0 \pi^2 a_e^2} \frac{1}{r} \exp(-\frac{r}{a_e}) \sin\theta \qquad (4.7)$$

If we combine equation (4.7) and (2.1), then we will get the Gauss Law of Electric Field.

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}$$

Thus we have proved that the electric field equation (4.2) indeed satisfies the Gauss Law of Electric Field.

Chapter

# 5 The magnetic field inside of the electron

Similar to the electric field, the Gauss Law of Magnetic Field has defined the relationship between magnetic charge and magnetic field. The Gauss Law of Magnetic Field is:

$$\nabla \cdot \vec{H} = \frac{\rho_m}{\mu_0} \tag{5.1}$$

In which $\vec{H}$ is the magnetic field strength, $\rho_m$ is the magnetic charge density.

Combine the Gauss Law of Magnetic Field (5.1) and magnetic charge density distribution equation (3.1), and we get the follows magnetic field strength solution:

$$\vec{H} = \frac{g}{\pi\mu_0 a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{5.2}$$

We will prove that the above equation (5.2) satisfies the Gauss Law of Magnetic Field. As we know the gradient $\nabla$ in spherical coordinate is:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial\phi}$$

From equation (5.2), thus:

$$\nabla \cdot \vec{H} = \frac{g}{\mu_0 \pi a_e}\{\frac{\partial}{\partial r}[\frac{1}{r}\exp(-\frac{r}{a_e})]\cos\theta + \frac{1}{r^2}\exp(-\frac{r}{a_e})\frac{\partial}{\partial\theta}\sin\vartheta]\} \tag{5.3}$$

Therefore,

$$\nabla \cdot \vec{H} = \frac{g}{\mu_0 \pi a_e}\{[-\frac{1}{r^2}\exp(-\frac{r}{a_e}) - \frac{1}{ra_e}\exp(-\frac{r}{a_e})]\cos\theta + \frac{1}{r^2}\exp(-\frac{r}{a_e})\cos\vartheta]\} \tag{5.4}$$

Thus:

$$\nabla \cdot \vec{H} = -\frac{g}{\mu_0 \pi a^2}\frac{1}{r}\exp(-\frac{r}{a_e})\cos\theta \tag{5.5}$$

Combine equation (3.1) and (5.5), then we get:

$$\nabla \cdot \vec{H} = \frac{\rho_m}{\mu_0}$$

Thus we have proved that the magnetic field solution equation (5.2) satisfied the Gauss Law of Magnetic Field.

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Chapter

# 6 The electromagnetic field angular momentum of the electron

As we know, the electromagnetic field has the properties of energy, momentum and angular momentum. The electromagnetic field momentum density is:

$$\vec{p} = \frac{1}{c^2}\vec{E}\times\vec{H} \tag{6.1}$$

Based upon the electromagnetic field momentum, and then we can define the field angular momentum density as follows:

$$\vec{\varsigma} = \vec{r}\times\vec{p} \tag{6.2}$$

Thus:

$$\vec{\varsigma} = \frac{1}{c^2}\vec{r}\times(\vec{E}\times\vec{H}) \tag{6.3}$$

Then integrating equation (6.3), we can get the field angular momentum:

$$\vec{L} = \int \frac{1}{c^2}\vec{r}\times(\vec{E}\times\vec{H})dr^3 \tag{6.4}$$

Combine the equation (4.2) and (5.2) into equation (6.3),
Thus:

$$\vec{\varsigma} = -\frac{eg}{\pi^3 a_e^2}\frac{1}{r}\exp(-\frac{2r}{a_e})\hat{\theta} \tag{6.5}$$

For the cylindrical coordinate ( $\rho$ , $\phi$ , z), which has the follows relationship with the spherical coordinate (r, $\phi$, $\theta$):

$$\rho = r\sin\theta$$

$$z = r\cos\theta$$

We can separate the angular momentum density into the z component and $\rho$ component,
The z component of angular momentum density is:

$$\varsigma_z = \frac{eg}{\pi^3 a_e^2}\frac{1}{r}\exp(-\frac{2r}{a_e})\sin\theta \tag{6.6}$$

The $\rho$ component of angular momentum density is:

$$\varsigma_\rho = -\frac{eg}{\pi^3 a_e^2}\frac{1}{r}\exp(-\frac{2r}{a_e})\cos\theta \tag{6.7}$$

As we know the volume element is: $dr^3 = 2\pi r^2 \sin\theta d\theta dr$
For the $\rho$ component of angular momentum density, because

$$\int_{\theta=0}^{\pi}\cos\theta\sin\theta d\theta = 0 \tag{6.8}$$

Thus we can get the $\rho$ component of angular momentum:

$$L_\rho = 0$$

For the z component of electron angular momentum, we have

$$L_z = \int \frac{eg}{\pi^3 a_e^2}\frac{1}{r}\exp(-\frac{2r}{a_e})\sin\theta 2\pi r^2 \sin\theta d\theta dr \tag{6.9}$$

Thus:

$$L_z = \frac{2eg}{\pi^2 a_e^2}\int r\exp(-\frac{2r}{a_e})dr\sin^2\theta d\theta \tag{6.10}$$

$$L_z = \frac{2eg}{\pi^2 a_e^2}\int_0^r r\exp(-\frac{2r}{a_e})dr\int_0^\theta \sin^2\theta d\theta \tag{6.11}$$

Therefore:

$$L_z(r,\theta) = \frac{eg}{4\pi^2}[1-(1+\frac{2r}{a_e})\exp(-\frac{2r}{a_e})](\theta - \frac{1}{2}\sin 2\theta) \tag{6.12}$$

The equation (6.12) is the angular momentum distribution equation of an electron.

When $\theta = \pi$, the z component of angular momentum is:

$$L_z(r,\pi) = \frac{eg}{4\pi}[1-(1+\frac{2r}{a_e})\exp(-\frac{2r}{a_e})] \tag{6.13}$$

The equation (6.13) is the angular momentum within the sphere of radius r of an electron.

The angular momentum distribution is a kind of cumulative gamma distribution in mathematics [B]

When $r \to \infty$ thus:

$$L_z = \frac{eg}{4\pi} \tag{6.14}$$

The above angular momentum is the electron's electromagnetic field angular momentum in total.

What is the electron spin? As we know, the electron spin is the electron intrinsic angular momentum. Let us make an assumption that electron spin is the electromagnetic field angular momentum, which also means that the electron spin is of purely electromagnetic origin.

Thus:

$$L_z = \frac{\hbar}{2} \tag{6.15}$$

Combine (6.14) and (6.15), thus:

$$eg = h \tag{6.16}$$

From equation (6.16), we found out that the multiple of electric charge unit 'e' and magnetic charge unit 'g' equal the Planck's constant 'h'.

Then, we can also calculate the ratio of magnetic charge 'g' and electric charge 'e' as follows:

$$\frac{g}{e} = \frac{h}{e^2} \tag{6.17}$$

Thus:

$$\frac{g}{e} = \frac{2\varepsilon_0 hc}{2\varepsilon_0 ce^2} \tag{6.18}$$

So:

$$\frac{g}{e} = \frac{1}{2\varepsilon_0 c\alpha} \tag{6.19}$$

Thus:

$$\frac{g}{e} = \frac{1}{2\alpha}\sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (6.20)$$

As we know, the vacuum impedance is:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (6.21)$$

So we can see the ratio of magnetic charge unit 'g' and electric charge unit 'e' has the unit of impedance.

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Chapter

# 7 The electric field energy of the electron

As we know, the electric field energy density is:

$$u_e = \frac{1}{2}\varepsilon_0 E^2 \tag{7.1}$$

Integrating energy density for the sphere of radius r, we can get the electric field energy:

$$U_e = \int \frac{1}{2}\varepsilon_0 E^2 dr^3 \tag{7.2}$$

With the electric field strength equation (4.2), then we can get:

$$U_e = \int \frac{e^2}{2\pi^4 \varepsilon_0 a_e^2} \frac{1}{r^2} \exp(-\frac{2r}{a_e}) 4\pi r^2 dr \tag{7.3}$$

Thus:

$$U_e(r) = \frac{e^2}{\pi^3 \varepsilon_0 a_e}[1 - \exp(-\frac{2r}{a_e})] \tag{7.4}$$

Equation (7.4) is the electric energy distribution equation of electron. We find out that the electric field energy has the spherical exponential distribution inside of the electron.

When $r \to \infty$, thus:

$$U_e = \frac{e^2}{\pi^3 \varepsilon_0 a_e} \tag{7.5}$$

Equation (7.5) is the electron's electric field total energy equation.

Chapter

# 8 The magnetic field energy of the electron

As we know, the magnetic field energy density is:

$$u_m = \frac{1}{2}\mu_0 H^2 \tag{8.1}$$

Integrating energy density to volume of the sphere of radius r, from the magnetic field equation (5.2), then we can get the electron magnetic field energy

$$U_m = \int \frac{2g^2}{\pi\mu_0 a_e^2}\exp(-\frac{2r}{a_e})dr \tag{8.2}$$

Thus:

$$U_m(r) = \frac{g^2}{\pi\mu_0 a_e}[1-\exp(-\frac{2r}{a_e})] \tag{8.3}$$

The equation (8.3) is the magnetic field energy within the sphere of the radius r. From equation (8.3), We find out that the magnetic field energy of electron has the spherical exponential distribution.

When $r \to \infty$, thus:

$$U_m = \frac{g^2}{\pi\mu_0 a_e} \tag{8.4}$$

Equation (8.4) is the electron magnetic field energy in total.

Chapter

# 9 The electromagnetic field energy of the electron

As we know, the electromagnetic field energy density is:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \tag{9.1}$$

Thus the electron electromagnetic field energy is:

$$U = U_e + U_m \tag{9.2}$$

Combine equation (7.5) and (8.4), thus:

$$\frac{U_e}{U_m} = \frac{e^2}{\pi^3 \varepsilon_0 a_e} \frac{\pi \mu_0 a_e}{g^2} \tag{9.3}$$

Thus:

$$\frac{U_e}{U_m} = \frac{e^2 \mu_0}{\pi^2 g^2 \varepsilon_0} \tag{9.4}$$

As we know:

$$ge = h \tag{9.5}$$

And also:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{9.6}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{9.7}$$

Thus:

$$\frac{U_e}{U_m} = (\frac{2\alpha}{\pi})^2 \tag{9.8}$$

Thus:

$$U = \frac{g^2}{\pi\mu_0 a_e}(1 + \frac{4\alpha^2}{\pi^2}) \tag{9.9}$$

Thus:

$$U = \frac{\hbar c}{a_e \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{9.10}$$

As we know, the electron mass is one of the basic properties of electrons. Where does the electron mass come from? One of the most popular theories is the electromagnetic origin of electron mass.

From the above equation (9.10), we get the electron's electromagnetic energy; let us make an assumption here that the electron's mass origin is from the electromagnetic field energy, thus:

$$U = m_e c^2 \tag{9.11}$$

In which U is the electron electromagnetic field energy, and $m_e$ is the electron mass, then combine the equation (9.10) and (9.11), we can get

$$m_e c^2 = \frac{\hbar c}{a_e \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{9.12}$$

Thus:

$$a_e = \frac{\hbar}{m_e c \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{9.13}$$

And we know that the Bohr radius is:

$$a_0 = \frac{\hbar}{m_e c \alpha} \tag{9.14}$$

Thus:

$$a_e = a_0 (1 + \frac{4\alpha^2}{\pi^2}) \tag{9.15}$$

As we know, the value of fine structure constant is about:

$$\alpha \approx \frac{1}{137} \tag{9.16}$$

Thus:

$$(\frac{2\alpha}{\pi})^2 << 1 \tag{9.17}$$

Thus:

$$a_0 \approx a_e \tag{9.18}$$

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Chapter

# 10 The velocity of the electromagnetic field

As we know, the velocity $\vec{V}$ , the mass m, and the momentum $\vec{P}$ has the follows relationship:

$$\vec{V} = \frac{\vec{P}}{m} \tag{10.1}$$

Let us make the assumption that the above relationship is also valid within the electromagnetic field. As we know for the electromagnetic field, the momentum density is:

$$\vec{p} = \frac{1}{c^2}\vec{E}\times\vec{H} \tag{10.2}$$

Per our assumption, the mass origin is from the electromagnetic field energy, so the mass density inside the electromagnetic field is:

$$\rho = \frac{u}{\mathrm{c}^2} \tag{10.3}$$

In which u is the electromagnetic field energy density:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \tag{10.4}$$

Thus, the velocity of electromagnetic field is as follows:

$$\vec{V} = \frac{\vec{p}}{\rho} \tag{10.5}$$

Thus:

$$\vec{V} = \frac{\vec{E}\times\vec{H}}{\frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2)} \tag{10.6}$$

As we know:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Thus:

$$\frac{\vec{V}}{c} = \frac{(\sqrt{\varepsilon_0}\vec{E}) \times (\sqrt{\mu_0}\vec{H})}{\frac{1}{2}[(\sqrt{\varepsilon_0}E)^2 + (\sqrt{\mu_0}H)^2]} \tag{10.7}$$

Equation (10.7) is the electromagnetic field velocity equation.

From the equation (10.7), we found out that the value of velocity $\vec{V}$ is always less than the speed of light c, when and only when both of the follows two conditions are satisfied, the velocity will equal to speed of light, which are:

$$\vec{E} \cdot \vec{H} = 0 \tag{10.8}$$

And

$$\sqrt{\varepsilon_0}E = \sqrt{\mu_0}H \tag{10.9}$$

Thus:

$$V = c \tag{10.10}$$

Combine the equations (4.2) and (5.2) into (10.7), then we can get the velocity of electromagnetic field inside the electron as follows:

$$\vec{V} = \frac{\frac{4}{\pi}c\alpha}{1 + (\frac{2}{\pi}\alpha)^2}\hat{\phi} \tag{10.11}$$

In which $\phi$ is the azimuthal angle.

From the above equation, we find out that inside of the electron, the velocity is around the azimuthal direction, and that the speed inside of the electron has constant value.

As we know:

$$(\frac{2\alpha}{\pi})^2 << 1$$

Thus:

$$\vec{V} \approx \frac{4c\alpha}{\pi}\hat{\phi} \tag{10.12}$$

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Chapter

# 11 The conservation of electric charge

The electric charge conservation is one of the basic conservation law in physics.
The electric charge conservation law is as follows:

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_e \vec{V}) = 0 \tag{11.1}$$

In which $\rho_e$ is the electric charge density.

Let us prove that the electric charge conservation equation is valid for the electron.

Combine the electron velocity equation (10.11) and the electric field equation (4.2), then we can get:

$$\vec{V} \times \vec{E} = \frac{Ve}{\pi^2 \varepsilon_0 a_e} \frac{1}{r} \exp(-\frac{r}{a_e})(\hat{\theta} \sin\theta + \hat{r} \cos\theta) \tag{11.2}$$

As we know:

$$\nabla = \hat{r} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\theta} \frac{\partial}{\partial \theta} + \frac{1}{r \sin\theta} \hat{\phi} \frac{\partial}{\partial \phi} \tag{11.3}$$

Thus:

$$\nabla \times (\vec{V} \times \vec{E}) = -\frac{eV\hat{\phi}}{\pi^2 \varepsilon_0 a_e^2} \frac{1}{r} \exp(-\frac{r}{a}) \sin\theta \tag{11.4}$$

From the electric charge density equation (2.1), thus:

$$\nabla \times (\vec{V} \times \vec{E}) = \frac{1}{\varepsilon_0} \rho_e \vec{V} \tag{11.5}$$

From equation (4.2), we know that:

$$\frac{\partial \vec{E}}{\partial t} = 0 \tag{11.6}$$

Thus, we can get

$$\varepsilon_0 \nabla \times (\vec{V} \times \vec{E}) = \rho_e \vec{V} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{11.7}$$

Compare this with the hydromagnetic equation (dynamo equation), which is as follows:

$$\nabla \times (\vec{V} \times \vec{B}) = -\lambda \nabla^2 \vec{B} + \frac{\partial \vec{B}}{\partial t} \tag{11.8}$$

In which $\lambda$ is the constant of the magnetic diffusivity.

We can see that there are many similarities between these two equations; we then call the equation (11.7) as the hydroelectric equation of electromagnetic field.

Because:

$$\nabla \cdot [\nabla \times (\vec{V} \times \vec{E})] = 0 \tag{11.9}$$

Thus:

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_e \vec{V}) = 0 \tag{11.10}$$

Equation (11.10) is the electron electric charge conservation equation, which is also called the electric charge continuity equation.

Chapter

# 12 The conservation of magnetic charge

Similar to the electric charge, the magnetic charge conservation law is as follows:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{V}) = 0 \tag{12.1}$$

In which $\rho_m$ is the magnetic charge density.

Let us prove that the magnetic charge conservation equation is valid for the electron.

Combine the electron velocity equation (10.11) and the magnetic field equation (5.2), then we get:

$$\vec{V} \times \vec{H} = \frac{Vg}{\pi \mu_0 a_e} \frac{1}{r} \exp(-\frac{r}{a_e})(\hat{\theta}\cos\theta - \hat{r}\sin\theta) \tag{12.2}$$

As we know:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial \phi} \tag{12.3}$$

Thus:

$$\nabla \times (\vec{V} \times \vec{H}) = -\frac{gV\hat{\phi}}{\pi \mu_0 a_e^2} \frac{1}{r} \exp(-\frac{r}{a})\cos\theta \tag{12.4}$$

From the magnetic charge density equation (3.1), thus:

$$\nabla \times (\vec{V} \times \vec{H}) = \frac{1}{\mu_0} \rho_m \vec{V} \tag{12.5}$$

From equation (5.2) we know that:

$$\frac{\partial \vec{H}}{\partial t} = 0 \tag{12.6}$$

Thus, we can get:

$$\mu_0 \nabla \times (\vec{V} \times \vec{H}) = \rho_m \vec{V} + \mu_0 \frac{\partial \vec{H}}{\partial t} \tag{12.7}$$

Compare with the hydromagnetic equation (dynamo equation), which is as follows:

$$\nabla \times (\vec{V} \times \vec{B}) = -\lambda \nabla^2 \vec{B} + \frac{\partial \vec{B}}{\partial t} \tag{12.8}$$

We can see there is much similarity between these two equations. We then call the equation (12.7) as the hydromagnetic equation of electromagnetic field.

Because:

$$\nabla \cdot [\nabla \times (\vec{V} \times \vec{H})] = 0 \tag{12.9}$$

Thus:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{V}) = 0 \tag{12.10}$$

Equation (12.10) is the electron magnetic charge conservation equation, which is also called the magnetic charge continuity equation.

Chapter

# 13 The electromagnetic field equation

From our electron model, we find out that the electromagnetic field has satisfied the 5 equations (4.1), (5.1), (10.7), (11.7), (12.7), here we rewrite each one of the equations as follows:

The Gauss Law of Electric Field:

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{13.1}$$

The Gauss Law of Magnetic Field:

$$\nabla \cdot \vec{H} = \frac{\rho_m}{\mu_0} \tag{13.2}$$

The velocity of electromagnetic field equation:

$$\frac{\vec{V}}{c} = \frac{(\sqrt{\varepsilon_0}\vec{E}) \times (\sqrt{\mu_0}\vec{H})}{\frac{1}{2}[(\sqrt{\varepsilon_0}E)^2 + (\sqrt{\mu_0}H)^2]} \tag{13.3}$$

The hydroelectric equation:

$$\varepsilon_0 \nabla \times (\vec{V} \times \vec{E}) = \rho_e \vec{V} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{13.4}$$

The hydromagnetic equation:

$$\mu_0 \nabla \times (\vec{V} \times \vec{H}) = \rho_m \vec{V} + \mu_0 \frac{\partial \vec{H}}{\partial t} \tag{13.5}$$

Let us consider one special case based on the above 5 electromagnetic field equations, the free charge electromagnetic field in vacuum, which satisfies the following four conditions:

$$\rho_e = 0 \tag{13.6}$$

$$\rho_m = 0 \quad (13.7)$$

$$\vec{E} \cdot \vec{H} = 0 \quad (13.8)$$

$$\sqrt{\varepsilon_0} E = \sqrt{\mu_0} H \quad (13.9)$$

Then, based on above 4 conditions, we get:

$$V = c \quad (13.10)$$

$$\varepsilon_0 \vec{V} \times \vec{E} = \sqrt{\varepsilon_0 \mu_0} c\vec{H} \quad (13.11)$$

Thus:

$$\varepsilon_0 \vec{V} \times \vec{E} = \vec{H} \quad (13.12)$$

And:

$$\mu_0 \vec{V} \times \vec{H} = -\sqrt{\varepsilon_0 \mu_0} c\vec{E} \quad (13.13)$$

Thus:

$$\mu_0 \vec{V} \times \vec{H} = -\vec{E} \quad (13.14)$$

Thus, we can get the following 5 equations of the electromagnetic field in vacuum of free charge, which are:

$$\nabla \cdot \vec{E} = 0 \quad (13.15)$$

$$\nabla \cdot \vec{H} = 0 \quad (13.16)$$

V=c (13.17)

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (13.18)$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (13.19)$$

Then based on the above 5 equations, we can get the Maxwell Electromagnetic Wave Equation in vacuum of free charge:

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \tag{13.20}$$

$$\nabla^2 \vec{H} = \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} \tag{13.21}$$

Chapter

# 14 The electromagnetic field equation in complex form

The electric field and magnetic field can be expressed in complex form.

Let us define the electromagnetic charge density in complex form as:

$$\rho = \frac{\rho_m}{\sqrt{\mu_0}} + i\frac{\rho_e}{\sqrt{\varepsilon_0}} \tag{14.1}$$

The Electromagnetic field strength in complex form as:

$$\vec{\Gamma} = \sqrt{\mu_0}\vec{H} + i\sqrt{\varepsilon_0}\vec{E} \tag{14.2}$$

And also:

$$\vec{\Gamma}^* = \sqrt{\mu_0}\vec{H} - i\sqrt{\varepsilon_0}\vec{E} \tag{14.3}$$

As we know the electromagnetic field energy density is:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \tag{14.4}$$

Thus, we have:

$$u = \frac{1}{2}\vec{\Gamma}\cdot\vec{\Gamma}^* \tag{14.5}$$

The electromagnetic field momentum density is:

$$\vec{p} = \frac{1}{c^2}\vec{E}\times\vec{H} \tag{14.6}$$

Thus, we have:

$$\vec{p} = \frac{1}{i2c}\vec{\Gamma}\times\vec{\Gamma}^* \tag{14.7}$$

Then we can rewrite the electromagnetic field equation as follows:

$$\nabla \cdot \Gamma = \rho \tag{14.8}$$

$$\vec{V} = \frac{c}{i}\frac{\Gamma \times \Gamma^*}{\Gamma \cdot \Gamma^*} \tag{14.9}$$

$$\nabla \times (V \times \vec{\Gamma}) = \rho \vec{V} + \frac{\partial \vec{\Gamma}}{\partial t} \tag{14.10}$$

The equation (14.10) can be called the hydro-electromagnetic equation.

As we know:

$$\nabla \times (V \times \vec{\Gamma}) = (\vec{\Gamma} \cdot \nabla)\vec{V} - (\vec{V} \cdot \nabla)\vec{\Gamma} + \vec{V}(\nabla \cdot \vec{\Gamma}) - \vec{\Gamma}(\nabla \cdot \vec{V}) \tag{14.11}$$

Thus:

$$\nabla \times (V \times \vec{\Gamma}) = (\vec{\Gamma} \cdot \nabla)\vec{V} - (\vec{V} \cdot \nabla)\vec{\Gamma} + \rho \vec{V} - \vec{\Gamma}(\nabla \cdot \vec{V}) \tag{14.12}$$

Thus the hydro-electromagnetic equation can be rewritten in the complex form as:

$$\frac{\partial \vec{\Gamma}}{\partial t} = (\vec{\Gamma} \cdot \nabla)\vec{V} - (\vec{V} \cdot \nabla)\vec{\Gamma} - \vec{\Gamma}(\nabla \cdot \vec{V}) \tag{14.13}$$

And thus, we can rewrite the equation (14.13) as follows:

$$\frac{\partial \vec{\Gamma}}{\partial t} + (\vec{V} \cdot \nabla)\vec{\Gamma} + \vec{\Gamma}(\nabla \cdot \vec{V}) = (\vec{\Gamma} \cdot \nabla)\vec{V} \tag{14.14}$$

Then the hydroelectric equation can be rewritten as:

$$\frac{\partial \vec{E}}{\partial t} + (\vec{V} \cdot \nabla)\vec{E} + \vec{E}(\nabla \cdot \vec{V}) = (\vec{E} \cdot \nabla)\vec{V} \tag{14.15}$$

Then the hydromagnetic equation can be rewritten as:

$$\frac{\partial \vec{H}}{\partial t} + (\vec{V} \cdot \nabla)\vec{H} + \vec{H}(\nabla \cdot \vec{V}) = (\vec{H} \cdot \nabla)\vec{V} \tag{14.16}$$

For the electron, we have

$$\rho_e(r,\theta) = -\frac{e}{\pi^2 a_e^{\,2}}\frac{1}{r}\exp(-\frac{r}{a_e})\sin\theta \tag{14.17}$$

And

$$\rho_m(r,\theta) = -\frac{g}{\pi a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e})\cos\theta \tag{14.18}$$

As we know:

$$\frac{g}{e}\sqrt{\frac{\varepsilon_0}{\mu_0}} = \frac{1}{2\alpha} \tag{14.19}$$

Thus:

$$\rho_m(r,\theta) = -\frac{\pi}{2\alpha}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{e}{\pi^2 a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e})\cos\theta \tag{14.20}$$

Let us define:

$$\rho_\alpha = \frac{\rho_m}{\sqrt{\mu_0}} + i\frac{\pi}{2\alpha}\frac{\rho_e}{\sqrt{\varepsilon_0}} \tag{14.21}$$

Thus:

$$\rho_\alpha = -\frac{\pi}{2\alpha}\frac{1}{\sqrt{\varepsilon_0}}\frac{e}{\pi^2 a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e} + i\theta) \tag{14.22}$$

So:

$$\rho_\alpha = -\frac{g}{\sqrt{\mu_0}}\frac{1}{\pi a_e^2}\frac{1}{r}\exp(-\frac{r}{a_e} + i\theta) \tag{14.23}$$

Let us define:

$$\vec{\Gamma}_\alpha = \sqrt{\mu_0}\vec{H} + i\frac{\pi}{2\alpha}\sqrt{\varepsilon_0}\vec{E} \tag{14.24}$$

As we know:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0 a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{14.25}$$

$$\vec{H} = \frac{g}{\pi\mu_0 a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{14.26}$$

Thus:

$$\sqrt{\varepsilon_0}\vec{E} = \frac{e}{\pi^2\sqrt{\varepsilon_0}a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{14.27}$$

$$\sqrt{\mu_0}\vec{H} = \frac{g}{\pi\sqrt{\mu_0}a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{14.28}$$

$$\sqrt{\mu_0}\vec{H} = \frac{\pi}{2\alpha}\frac{e}{\pi^2\sqrt{\varepsilon_0}a_e}\frac{1}{r}\exp(-\frac{r}{a_e})(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{14.29}$$

Thus:

$$\vec{\Gamma}_\alpha = \frac{\pi}{2\alpha}\frac{e}{\pi^2\sqrt{\varepsilon_0}a_e}\frac{1}{r}\exp(-\frac{r}{a_e} + i\theta)(\hat{r} - i\hat{\theta}) \tag{14.30}$$

Thus:

$$\vec{\Gamma}_\alpha = \frac{g}{\sqrt{\mu_0}}\frac{1}{\pi a_e r}\exp(-\frac{r}{a_e} + i\theta)(\hat{r} - i\hat{\theta}) \tag{14.31}$$

Chapter

# 15 The conservation of energy

The conservation of energy is one of the most basic physics laws, for the electromagnetic field, the energy conservation equation is as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (15.1)$$

In which $\rho$ is the mass density, $\vec{V}$ is the field velocity.
The mass density and energy density have the following relationship:

$$u = \rho c^2$$

In which u is the electromagnetic field energy density.

From equation (14.15) and (14.16), we have:

$$\frac{\partial \vec{E}}{\partial t} + (\vec{V} \cdot \nabla)\vec{E} + \vec{E}(\nabla \cdot \vec{V}) = (\vec{E} \cdot \nabla)\vec{V} \quad (15.2)$$

And

$$\frac{\partial \vec{H}}{\partial t} + (\vec{V} \cdot \nabla)\vec{H} + \vec{H}(\nabla \cdot \vec{V}) = (\vec{H} \cdot \nabla)\vec{V} \quad (15.3)$$

Thus:

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \vec{E} \cdot [(\vec{V} \cdot \nabla)\vec{E}] + E^2(\nabla \cdot \vec{V}) = \vec{E} \cdot [(\vec{E} \cdot \nabla)\vec{V}] \quad (15.4)$$

And

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} + \vec{H} \cdot [(\vec{V} \cdot \nabla)\vec{H}] + H^2(\nabla \cdot \vec{V}) = \vec{H} \cdot [(\vec{H} \cdot \nabla)\vec{V}] \quad (15.5)$$

Thus:

$$\frac{1}{2}\frac{\partial E^2}{\partial t}+\frac{1}{2}[(\vec{V}\cdot\nabla)E^2]+E^2(\nabla\cdot\vec{V})=\vec{E}\cdot[(\vec{E}\cdot\nabla)\vec{V}] \tag{15.6}$$

And

$$\frac{1}{2}\frac{\partial H^2}{\partial t}+\frac{1}{2}[(\vec{V}\cdot\nabla)H^2]+H^2(\nabla\cdot\vec{V})=\vec{H}\cdot[(\vec{H}\cdot\nabla)\vec{V}] \tag{15.7}$$

Combine (15.6) and (15.7), thus:

$$\frac{\partial u}{\partial t}+(\vec{V}\cdot\nabla)u+2u(\nabla\cdot\vec{V})=\varepsilon_0\vec{E}\cdot[(\vec{E}\cdot\nabla)\vec{V}]+\mu_0\vec{H}\cdot[(\vec{H}\cdot\nabla)\vec{V}] \tag{15.8}$$

Let us assume that the electromagnetic energy is conserved, therefore, we have:

$$\frac{\partial u}{\partial t}+\nabla\cdot(u\vec{V})=0 \tag{15.9}$$

Thus:

$$\frac{\partial u}{\partial t}+\vec{V}\cdot\nabla u+u(\nabla\cdot\vec{V})=0 \tag{15.10}$$

Thus, we have:

$$u(\nabla\cdot\vec{V})=\varepsilon_0\vec{E}\cdot[(\vec{E}\cdot\nabla)\vec{V}]+\mu_0\vec{H}\cdot[(\vec{H}\cdot\nabla)\vec{V}] \tag{15.11}$$

As we know:

$$u_e=\frac{1}{2}\varepsilon_0E^2 \tag{15.12}$$

$$u_m=\frac{1}{2}\mu_0H^2 \tag{15.13}$$

Let us define:

$$\vec{E}=E\hat{n}_e \tag{15.14}$$

$$\vec{H}=H\hat{n}_m \tag{15.15}$$

From the electromagnetic field energy conservation equation, then we can get the following equation:

$$(\nabla\cdot\vec{V})=\frac{2u_e}{u}\hat{n}_e\cdot[(\hat{n}_e\cdot\nabla)\vec{V}]+\frac{2u_m}{u}\hat{n}_m\cdot[(\hat{n}_m\cdot\nabla)\vec{V}] \tag{15.16}$$

Then we can rewrite the electromagnetic field as follows:

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{15.17}$$

$$\nabla \cdot \vec{H} = \frac{\rho_m}{\mu_0} \tag{15.18}$$

$$\frac{\vec{V}}{c} = \frac{(\sqrt{\varepsilon_0}\vec{E}) \times (\sqrt{\mu_0}\vec{H})}{\frac{1}{2}[(\sqrt{\varepsilon_0}E)^2 + (\sqrt{\mu_0}H)^2]} \tag{15.19}$$

$$\frac{\partial \vec{E}}{\partial t} + (\vec{V} \cdot \nabla)\vec{E} + \vec{E}(\nabla \cdot \vec{V}) = (\vec{E} \cdot \nabla)\vec{V} \tag{15.20}$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{V} \cdot \nabla)\vec{H} + \vec{H}(\nabla \cdot \vec{V}) = (\vec{H} \cdot \nabla)\vec{V} \tag{15.21}$$

$$\nabla \cdot \vec{V} = \frac{2u_e}{u}\hat{n}_e \cdot [(\hat{n}_e \cdot \nabla)\vec{V}] + \frac{2u_m}{u}\hat{n}_h \cdot [(\hat{n}_h \cdot \nabla)\vec{V}] \tag{15.22}$$

Chapter

# 16 The electromagnetic model of the proton

As we know the atom consisted of electrons and nucleus, and the nucleus consists of protons and neutrons.

Both protons and neutrons have the half spin just like electrons do. But the proton has one positive electric charge unit 'e' and the neutron as a whole has zero electric charge.

Comparing protons and electrons, the proton's mass is much larger than the electron's mass, but they have the same amount of electric charge (with different signs), protons have positive electric charges, and the electrons have negative electric charges, both the electrons and protons have the same value of half spin.

Let us make an assumption that the proton has a similar internal structure as electrons, the difference is that the proton has one positive electric charge, and the mass of the proton is much larger than that of the electron.

The electric charge density distribution inside the proton is as follows:

$$\rho_e(r,\theta) = \frac{e}{\pi^2 a_p^2}\frac{1}{r}\exp(-\frac{r}{a_p})\sin\theta \tag{16.1}$$

The magnetic charge density distribution inside the proton is as follows:

$$\rho_m(r,\theta) = \frac{g}{\pi a_p^2}\frac{1}{r}\exp(-\frac{r}{a_p})\cos\theta \tag{16.2}$$

Based on the above charge distribution equation, we can get the proton's electromagnetic field distribution equation as follows:

$$\vec{E} = -\frac{e}{\pi^2 \varepsilon_0 a_p}\frac{1}{r}\exp(-\frac{r}{a_p})(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{16.3}$$

$$\vec{H} = -\frac{g}{\pi \mu_0 a_p}\frac{1}{r}\exp(-\frac{r}{a_p})(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{16.4}$$

Based on the electromagnetic field distribution equation, similar to the electron, we can get the proton's electromagnetic field energy:

$$U = \frac{\hbar c}{a_p \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{16.5}$$

Then, we make the assumption similar to the electron that the proton mass also has electromagnetic origin, then we have:

$$m_p c^2 = \frac{\hbar c}{a_p \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{16.6}$$

Compare this with the electron energy formula:

$$m_e c^2 = \frac{\hbar c}{a_e \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \tag{16.7}$$

So we can get following relationship:

$$\frac{m_p}{m_e} = \frac{a_e}{a_p} \tag{16.8}$$

As we know that:

$$a_e = a_0[1 + (\frac{4\alpha}{\pi})^2] \tag{16.9}$$

Thus:

$$a_p = a_0[1 + (\frac{4\alpha}{\pi})^2]\frac{m_e}{m_p} \tag{16.10}$$

In within $a_0$ is the Bohr radius.

Chapter

# 17 The electromagnetic model of the hydrogen atom

Despite the hydrogen atom's overall neutrality, hydrogen does have an internal distribution of electric charge.

As we know that the simplest hydrogen atom consists of one proton and one electron, the hydrogen atom as total has zero electric charge. Combine both electron charge distribution and proton charge distribution together, then we can get the hydrogen atom's electric charge distribution as follows:

$$\rho_e = \frac{e}{\pi^2}[-\frac{1}{a_e}\exp(-\frac{r}{a_e}) + \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}\sin\theta \tag{17.1}$$

Hydrogen atom magnetic charge density distribution equation:

$$\rho_m = \frac{g}{\pi a^2}[-\frac{1}{a_e}\exp(-\frac{r}{a_e}) + \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}\cos\theta \tag{17.2}$$

Hydrogen atom electric charge distribution equation:

$$Q_e = e[-\exp(-\frac{r}{a_e})(1+\frac{r}{a_e}) + \exp(-\frac{r}{a_p})(1+\frac{r}{a_p})]\frac{1}{\pi}(\theta - \frac{1}{2}\sin 2\theta) \tag{17.3}$$

Hydrogen atom magnetic charge distribution equation:

$$Q_m = g[\exp(-\frac{r}{a_e})(1+\frac{r}{a_e}) - \exp(-\frac{r}{a_p})(1+\frac{r}{a_p})]\frac{1}{2}(1-\cos 2\theta) \tag{17.4}$$

Hydrogen atom electric field equation:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{17.5}$$

Hydrogen atom magnetic field equation:

$$\vec{H} = \frac{g}{\pi\mu_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{17.6}$$

Base on these equation, we can get hydrogen magnetic field energy:

$$U_m = \frac{2g^2}{\pi\mu_0}(\frac{1}{2a_e} + \frac{1}{2a_p} - \frac{2a_d}{a_e a_p}) \tag{17.7}$$

In which

$$\frac{1}{a_d} = \frac{1}{a_e} + \frac{1}{a_p} \tag{17.8}$$

Thus:

$$U_m = \frac{2g^2}{\pi\mu_0}(\frac{1}{2a_e} + \frac{1}{2a_p} - \frac{2}{(a_e + a_p)}) \tag{17.9}$$

Thus:

$$U_m = \frac{2g^2}{\pi\mu_0}(\frac{1}{2a_e} + \frac{1}{2a_p} - \frac{2}{(a_e + a_p)}) \tag{17.10}$$

Thus:

$$U_m = \frac{g^2}{\pi\mu_0}(\frac{1}{a_e} + \frac{1}{a_p} - \frac{4}{(a_e + a_p)}) \tag{17.11}$$

The hydrogen electric field energy is:

$$U_e = \frac{2e^2}{\pi^3\varepsilon_0}(\frac{1}{2a_e} + \frac{1}{2a_p} - \frac{2}{a_e + a_p}) \tag{17.12}$$

$$U_e = \frac{e^2}{\pi^3\varepsilon_0}(\frac{1}{a_e} + \frac{1}{a_p} - \frac{4}{a_e + a_p}) \tag{17.13}$$

Thus, the hydrogen atom’s electromagnetic energy is:

$$U_H = U_m + U_e \tag{17.14}$$

Thus:

$$U_H = U_{proton} + U_{electron} - 4U_{electron} \frac{a_e}{a_e + a_p} \quad (17.15)$$

As we know

$$\frac{a_p}{a_e} << 1 \quad (17.16)$$

Thus:

$$U_H \approx U_{proton} - 3U_{electron} \quad (17.17)$$

Thus:

$$m_H \approx m_p - 3m_e \quad (17.18)$$

As we know:

$$\vec{E} = \frac{e}{\pi^2 \varepsilon_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \quad (17.19)$$

$$\vec{H} = \frac{g}{\pi \mu_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \quad (17.20)$$

Thus:

$$\vec{E} \times \vec{H} = \frac{eg}{\pi^3 \varepsilon_0 \mu_0 r^2}\vec{\varphi}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \quad (17.21)$$

Thus, the momentum density for hydrogen is:

$$\vec{p} = \frac{eg}{\pi^3 r^2}\vec{\varphi}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \quad (17.22)$$

The energy density for hydrogen is:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \quad (17.23)$$

$$u = \frac{1}{2\pi^2}(\frac{e^2}{\pi^2 \varepsilon_0} + \frac{g^2}{\mu_0})\frac{1}{r^2}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \quad (17.24)$$

Thus, the velocity of a hydrogen atom is:

$$\vec{V} = \frac{2c^2 eg}{\pi} \frac{1}{(\frac{e^2}{\pi^2 \varepsilon_0} + \frac{g^2}{\mu_0})} \vec{\varphi} \tag{17.25}$$

$$\vec{V} = \frac{2e}{\pi g \varepsilon_0} \frac{1}{(1 + \frac{e^2 \mu_0}{g^2 \pi^2 \varepsilon_0})} \vec{\varphi} \tag{17.26}$$

Thus:

$$\vec{V} = \frac{\frac{4}{\pi} c\alpha}{1 + (\frac{2}{\pi}\alpha)^2} \hat{\phi} \tag{17.27}$$

We find out that inside the hydrogen atom, the velocity is the same as for the electron, which has a constant speed value.

As we know the field angular momentum density is:

$$\vec{\varsigma} = \frac{1}{c^2} \vec{r} \times (\vec{E} \times \vec{H}) \tag{17.28}$$

Thus, the hydrogen atom field angular momentum is as follows:

$$\vec{\varsigma} = -\frac{eg}{\pi^3 r} \vec{\theta} [\frac{1}{a_e} \exp(-\frac{r}{a_e}) - \frac{1}{a_P} \exp(-\frac{r}{a_P})]^2 \tag{17.29}$$

Then, integrating equation (17.28), we can get the field angular momentum:

$$\vec{L} = \int \frac{1}{c^2} \vec{r} \times (\vec{E} \times \vec{H}) dr^3 \tag{17.30}$$

$$\vec{\varsigma} = -\frac{eg}{\pi^3 r} \vec{\theta} [\frac{1}{a_e} \exp(-\frac{r}{a_e}) - \frac{1}{a_P} \exp(-\frac{r}{a_P})]^2 \tag{17.31}$$

For the cylindrical coordinates ($\rho$, $\phi$, z), which has the following relationship with the spherical coordinate (r, $\phi$, $\theta$):

$$\rho = r \sin\theta$$

$$z = r\cos\theta$$

We can separate the angular momentum density into z component and $\rho$ component, The z component of angular momentum density is:

$$\varsigma_z = \frac{eg}{\pi^3}\frac{1}{r}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \sin\theta \tag{17.32}$$

The $\rho$ component of angular momentum density is:

$$\varsigma_\rho = -\frac{eg}{\pi^3}\frac{1}{r}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \cos\theta \tag{17.33}$$

As we know, the volume element is: $dr^3 = 2\pi r^2 \sin\theta d\theta dr$
For the $\rho$ component of angular momentum density, because

$$\int_{\theta=0}^{\pi} \cos\theta \sin\theta d\theta = 0 \tag{17.34}$$

Thus, we can get the $\rho$ component of angular momentum:

$$L_\rho = 0 \tag{17.35}$$

For the z component of electron angular momentum, we have:

$$L_z = \int \frac{eg}{\pi^3}\frac{1}{r}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 \sin\theta 2\pi r^2 \sin\theta d\theta dr \tag{17.36}$$

Thus:

$$L_z = \frac{2eg}{\pi^2}\int r[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 dr \sin^2\theta d\theta \tag{17.37}$$

Thus:

$$L_z = \frac{2eg}{\pi^2}\int_0^r r[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 dr \int_0^\theta \sin^2\theta d\theta \tag{17.38}$$

Thus:

$$L_z = \frac{eg}{\pi^2}\int_0^r r[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 dr(\theta - \frac{1}{2}\sin 2\theta) \tag{17.39}$$

When $\theta = \pi$, thus the z component of angular momentum is:

$$L_z = \frac{eg}{\pi}\int_0^r r[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]^2 dr \tag{17.40}$$

When $r \to \infty$ thus:

$$L_z = \frac{eg}{\pi}\{\frac{1}{4}+\frac{1}{4}-\frac{2a_d^2}{a_e a_p}\} \tag{17.41}$$

In which

$$\frac{1}{a_d} = \frac{1}{a_e}+\frac{1}{a_p} \tag{17.42}$$

Thus:

$$L_z = \frac{eg}{2\pi}\{1-\frac{4a_d^2}{a_e a_p}\} \tag{17.43}$$

As we know eg= h

Thus:

$$L_z = \hbar\{1-\frac{4a_d^2}{a_e a_p}\} \tag{17.44}$$

Thus:

$$L = \hbar(1-4\frac{a_d^2}{a_e a_p}) \tag{17.45}$$

Thus:

$$L = \hbar(1-4\frac{a_e a_p}{(a_e+a_p)^2}) \tag{17.46}$$

Thus:

$$L = \hbar(1-4\frac{1}{(1+\frac{a_p}{a_e})^2}\frac{a_p}{a_e}) \tag{17.47}$$

As we know

$$\frac{a_p}{a_e} << 1 \tag{17.48}$$

Thus:

$$L \approx \hbar \tag{17.49}$$

Based on the equations (17.3) and (17.4), we can get the ratio of magnetic charge and electric charge inside the hydrogen atom:

$$\frac{Q_m(r,\theta)}{Q_e(r,\theta)} = -\frac{\pi g}{2e}\frac{(1-\cos 2\theta)}{(\theta - \frac{1}{2}\sin 2\theta)} \tag{17.50}$$

For the whole north pole area which $\theta$ is within $(0 \to \frac{\pi}{2})$

Thus:

$$\frac{Q_m(r, 0 \to \frac{\pi}{2})}{Q_e(r, 0 \to \frac{\pi}{2})} = -\frac{2g}{e} \tag{17.51}$$

For the whole south pole area which $\theta$ is within $(\frac{\pi}{2} \to \pi)$

Thus:

$$\frac{Q_m(r, \frac{\pi}{2} \to \pi)}{Q_e(r, \frac{\pi}{2} \to \pi)} = \frac{2g}{e} \tag{17.52}$$

Chapter

# 18 The electromagnetic model of the neutron

The neutron as a whole has zero electric charge and zero magnetic charge, but the neutron still has an intrinsic electric field and magnetic field.

The neutron in total has zero electric charge and magnetic charge, but similar to the electron and proton, the neutron still has half spin.

Let us compare the neutron and hydrogen atoms, both as a total have zero electric charge and zero magnetic charge, and both have intrinsic electric field and magnetic field. The major difference is the neutron alone has half spin.

Let us assume that a neutron has a similar internal structure to a hydrogen atom. Then we can make an assumption that the neutron's electric and magnetic charge distribution equation is as follows:

$$\rho_e = \frac{e}{\pi^2}\left[\frac{1}{a_+}\exp\left(-\frac{r}{a_+}\right) - \frac{1}{a_-}\exp\left(-\frac{r}{a_-}\right)\right]\frac{1}{r}\sin\theta \quad (18.1)$$

The neutron's magnetic charge density distribution equation is as follows:

$$\rho_m = \frac{g}{\pi a^2}\left[\frac{1}{a_+}\exp\left(-\frac{r}{a_+}\right) - \frac{1}{a_-}\exp\left(-\frac{r}{a_-}\right)\right]\frac{1}{r}\cos\theta \quad (18.2)$$

The neutron as whole has no electric charge, but has half spin. Here is the model for the neutron, which has a similar electromagnetic structure as a hydrogen atom. The difference is that the hydrogen atom has spin 1, but the neutron has half spin.

Similar to the hydrogen atom spin formula, we can get the neutron's angular momentum as follows:

$$L = \hbar\left(1 - 4\frac{\frac{a_+}{a_-}}{\left(1+\frac{a_+}{a_-}\right)^2}\right) \quad (18.3)$$

As we know neutrons have half spin, so:

$$L = \frac{\hbar}{2} \tag{18.4}$$

Let us make an assumption that

$$a_+ < a_-$$

Thus:

$$\frac{a_+}{a_-} = 3 - 2\sqrt{2} \tag{18.5}$$

Thus:

$$\frac{a_-}{a_+} = 3 + 2\sqrt{2} \tag{18.6}$$

We can get the neutron's magnetic field energy as follows:

$$U_m = \frac{2g^2}{\pi\mu_0}(\frac{1}{2a_+} + \frac{1}{2a_-} - \frac{2}{a_+ + a_-}) \tag{18.7}$$

Thus:

$$U_m = \frac{2g^2}{\pi\mu_0}\frac{(2+\sqrt{2})}{2a_+} \tag{18.8}$$

Thus:

$$U_m = \frac{g^2}{\pi\mu_0}\frac{(2+\sqrt{2})}{a_+} \tag{18.9}$$

Thus:

$$U_m = \frac{g^2}{\pi\mu_0}\frac{1}{a_n} \tag{18.10}$$

In which

$$a_n = a_+(1 - \frac{\sqrt{2}}{2}) \tag{18.11}$$

Thus:

$$a_+ = 2a_n(1+\frac{\sqrt{2}}{2}) \quad (18.12)$$

Therefore, we can get the neutron's electric field energy as follows:

$$U_e = \frac{2e^2}{\pi^3\varepsilon_0}(\frac{1}{2a_+}+\frac{1}{2a_-}-\frac{2}{a_+ + a_-}) \quad (18.13)$$

Thus:

$$U_e = \frac{2e^2}{\pi^3\varepsilon_0}\frac{(2+\sqrt{2})}{2a_+} \quad (18.14)$$

Thus:

$$U_e = \frac{e^2}{\pi^3\varepsilon_0}\frac{(2+\sqrt{2})}{a_+} \quad (18.15)$$

Thus:

$$U_e = \frac{e^2}{\pi^3\varepsilon_0}\frac{1}{a_n} \quad (18.16)$$

As we know, the electromagnetic field energy is:

$$U = U_m + U_e \quad (18.17)$$

Thus:

$$U = \frac{\hbar c}{a_n\alpha}(1+\frac{4\alpha^2}{\pi^2}) \quad (18.18)$$

Thus:

$$U = m_n c^2 \quad (18.19)$$

$m_n$ is the neutron mass, thus:

$$m_n c^2 = \frac{\hbar c}{a_n\alpha}(1+\frac{4\alpha^2}{\pi^2}) \quad (18.20)$$

Thus:

$$a_n = \frac{\hbar}{m_n c\alpha}(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.21)$$

Thus:

$$a_+ = a_n(2 + \sqrt{2}) \quad (18.22)$$

$$a_+ = \frac{\hbar}{m_n c\alpha}(2 + \sqrt{2})(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.23)$$

As we know,

$$\frac{a_-}{a_+} = 3 + 2\sqrt{2} \quad (18.24)$$

Thus:

$$a_- = \frac{\hbar}{m_n c\alpha}(3 + 2\sqrt{2})(2 + \sqrt{2})(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.25)$$

Thus:

$$a_- = \frac{\hbar}{m_n c\alpha}(10 + 7\sqrt{2})(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.26)$$

Compare this with the electron formula:

$$m_e c^2 = \frac{\hbar c}{a_e \alpha}(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.27)$$

Thus:

$$a_e = \frac{\hbar}{m_e c\alpha}(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.28)$$

As we know,

$$a_0 = \frac{\hbar}{m_e c\alpha} \quad (18.29)$$

Thus:

$$a_e = a_0(1 + \frac{4\alpha^2}{\pi^2}) \quad (18.30)$$

Thus, we get the relationship between the electron mass, neutron mass and their radius relationship:

$$\frac{a_-}{a_e} = (10 + 7\sqrt{2})\frac{m_e}{m_n} \tag{18.31}$$

$$\frac{a_+}{a_e} = (2 + \sqrt{2})\frac{m_e}{m_n} \tag{18.32}$$

# Chapter 19 The electromagnetic model of helium atom

The simplest helium atom consists of two protons and two electrons.
The helium atom's electric charge and magnetic charge density distribution is:

$$\rho_e = \frac{e}{\pi^2}[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) + \frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{p2}}\exp(-\frac{r}{a_{P2}}) - \frac{1}{a_{p1}}\exp(-\frac{r}{a_{P1}})]\frac{1}{r}\sin\theta \tag{19.1}$$

Helium's magnetic charge density distribution is:

$$\rho_m = \frac{g}{2\pi a^2}[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) + \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]\frac{1}{r}\cos\theta \tag{19.2}$$

The helium atom's electric charge distribution equation is:

$$Q_e = e[\exp(-\frac{r}{a_{e1}})(1+\frac{r}{a_{e1}}) + \exp(-\frac{r}{a_{e2}})(1+\frac{r}{a_{e2}}) - \exp(-\frac{r}{a_{p1}})(1+\frac{r}{a_{p1}}) - \exp(-\frac{r}{a_{p2}})(1+\frac{r}{a_{p2}})]\frac{1}{\pi}(\theta - \frac{1}{2}\sin 2\theta) \tag{19.3}$$

Helium's magnetic charge distribution equation is:

$$Q_m = g[\exp(-\frac{r}{a_{e1}})(1+\frac{r}{a_{e1}}) - \exp(-\frac{r}{a_{e2}})(1+\frac{r}{a_{e2}}) + \exp(-\frac{r}{a_{p1}})(1+\frac{r}{a_{p1}}) - \exp(-\frac{r}{a_{p2}})(1+\frac{r}{a_{p2}})]\frac{1}{2}(1-\cos 2\theta) \tag{19.4}$$

Helium's electric field equation is:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0}[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) + \frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]\frac{1}{r}(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{19.5}$$

The helium atom's magnetic field equation is:

$$\vec{H} = \frac{g}{\pi\mu_0}[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) + \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]\frac{1}{r}(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{19.6}$$

Thus:

$$\vec{E}\times\vec{H} = \frac{eg}{\pi^3\varepsilon_0\mu_0 r^2}\vec{\varphi}\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\} \tag{19.7}$$

As we know from (6.3), thus:

$$\vec{\varsigma} = \frac{1}{c^2}\vec{r} \times (\vec{E} \times \vec{H}) \qquad (19.8)$$

Thus:

$$\vec{\varsigma} = -\frac{eg}{\pi^3 r}\vec{\theta}\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\} \qquad (19.9)$$

For the cylindrical coordinates ( $\rho$ , $\phi$ , z), which has the following relationship with the spherical coordinate (r, $\phi$, $\theta$):

$$\rho = r\sin\theta$$

$$z = r\cos\theta$$

We can separate the angular momentum density into 'z' component and ' $\rho$ ' component,
The 'z' component of angular momentum density is:

$$\varsigma_z = \frac{eg}{\pi^3}\frac{1}{r}\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}\sin\theta \qquad (19.10)$$

The ' $\rho$ ' component of angular momentum density is:

$$\varsigma_\rho = -\frac{eg}{\pi^3}\frac{1}{r}\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}\cos\theta \qquad (19.11)$$

As we know, the volume element is: $dr^3 = 2\pi r^2 \sin\theta d\theta dr$
For the ' $\rho$ ' component of angular momentum density, because:

$$\int_{\theta=0}^{\pi}\cos\theta\sin\theta d\theta = 0$$

Thus, we can get the ' $\rho$ ' component of angular momentum:

$$L_\rho = 0 \qquad (19.12)$$

For the 'z' component of electron angular momentum, we have:

$$L_z = \int\frac{eg}{\pi^3}\frac{1}{r}\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}\sin\theta 2\pi r^2 \sin\theta d\theta dr \qquad (19.13)$$

Thus:

$$L_z = \frac{2eg}{\pi^2}\int r\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}dr\sin^2\theta d\theta \qquad (19.14)$$

Thus:

$$L_z = \frac{2eg}{\pi^2}\int_0^r r\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}dr\int_0^\theta \sin^2\theta d\theta \tag{19.15}$$

Thus:

$$L_z(r,\theta) = \frac{eg}{\pi^2}\int_0^r r\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}dr(\theta - \frac{1}{2}\sin 2\theta) \tag{19.16}$$

When $\theta = \pi$ , thus the z component of angular momentum is:

$$L_z(r,\pi) = \frac{eg}{\pi}\int_0^r r\{[\frac{1}{a_{e1}}\exp(-\frac{r}{a_{e1}}) - \frac{1}{a_{P2}}\exp(-\frac{r}{a_{P2}})]^2 - [\frac{1}{a_{e2}}\exp(-\frac{r}{a_{e2}}) - \frac{1}{a_{P1}}\exp(-\frac{r}{a_{P1}})]^2\}dr \tag{19.17}$$

When $r \to \infty$ , thus:

$$L_z(\infty,\pi) = \frac{2eg}{\pi}\{\frac{a_{e2p1}^2}{a_{e2}a_{p1}} - \frac{a_{e1p2}^2}{a_{e1}a_{p2}}\} \tag{19.18}$$

In which

$$\frac{1}{a_{e2p1}} = \frac{1}{a_{e2}} + \frac{1}{a_{p1}} \tag{19.19}$$

$$\frac{1}{a_{e1p2}} = \frac{1}{a_{e1}} + \frac{1}{a_{p2}} \tag{19.20}$$

As we know, eg=h

Thus:

$$L_z(\infty,\pi) = 4\hbar\{\frac{a_{p1}}{a_{e2}}\frac{1}{(1+\frac{a_{p1}}{a_{e2}})^2} - \frac{a_{p2}}{a_{e1}}\frac{1}{(1+\frac{a_{p2}}{a_{e1}})^2}\} \tag{19.21}$$

As we know:

$$\frac{a_{p1}}{a_{e2}} << 1 \tag{19.22}$$

And

$$\frac{a_{p2}}{a_{e1}} << 1 \tag{19.23}$$

Thus:

$$L_z(\infty, \pi) \approx 0 \tag{19.24}$$

The helium atom's spin (angular momentum) is almost zero.

Chapter

# 20

# The electromagnetic characteristic impedance

As we know, the Quantum Hall effect can be used to define the impedance unit. There are two kinds of Quantum Hall effect: Integer Quantum Hall effect and Fractional Quantum Hall effect. The integer Quantum Hall effect can be regarded as a special case of Fractional Quantum Hall effect.

For the Fractional Quantum Hall effect, in which the two-dimensional electron gas within extremely low temperature and high magnetic field perpendicular to current, the Hall Resistance has the following equation:

$$Z = \frac{m}{n}(\frac{h}{e^2}) \tag{20.1}$$

In which m and n are the integers.

As we know, the electron has electric charge and magnetic charge, the magnetic charge distributes along with the electron spin directional, and the electron spin will align with the external magnetic field. The one side of the two dimensional electron gases will have positive magnetic charge distribution; the other side of the two dimensional electron gases has negative magnetic charge distribution.

Let us make an assumption that the characteristic impedance is the ratio of magnetic charge and electric charge, then:

$$Z = \frac{Q_m}{Q_e} \tag{20.2}$$

As we know, the electron is the smallest unit of electron gas, the 'e' is the electric charge unit of the electron, and 'g' is the magnetic charge unit of an electron. Thus, we have:

$$Q_m = mg \tag{20.3}$$

And

$$Q_e = ne \tag{20.4}$$

Both 'm' and 'n' are integers.

Then we have

$$Z = \frac{m}{n}(\frac{g}{e}) \tag{20.5}$$

As we know:

$$ge = h \tag{20.6}$$

Thus, we have:

$$Z = \frac{m}{n}(\frac{h}{e^2}) \tag{20.7}$$

And so we get the result of the Fractional Quantum Hall effect.
When the ratio ($\frac{n}{m}$) becomes the integer, the Quantum Hall effect is the Integer Quantum Hall effect. The Integer Quantum Hall effect is the special case of the fractional quantum hall effect.

Chapter

# 21

## The electromagnetic wave

There are two kinds wave within the electromagnetic field, one is the field wave, and the other is the charge density wave.
According to the Gauss Law for both electric field and magnetic field, the electric charge is the source of electric field, and the magnetic charge is the source of the magnetic field.

When an electric current flows through a wire, it creates the electromagnetic field both inside and outside the wire.

Outside the electric wire is a vacuum, both the electric charge density and magnetic density in the vacuum is zero.

When the electric wire has ac current, there exists electromagnetic field wave propagate along the wire.

Inside the electric wire, the electric charge density wave and magnetic charge density charge wave is the source of the electromagnetic field wave.

Outside the wire is a vacuum, which has vacuum impedance. The electromagnetic field wave in the vacuum has light speed.

The electric wire has characteristic electromagnetic impedance. Both the charge density wave and field wave have the same wave velocity. The wave velocity depends on the characteristic impedance of the transmission line. The higher the impedance, the lower the velocity speed. The impedance of the transmission line and velocity of the charge density wave have the reversion relationship:

$$\frac{v}{c} = \frac{z_0}{z}$$

In which ‘v’ is the velocity of charge density sound wave, ‘c’ is the speed of light in vacuum, ‘ $z_0$ ’ is the impedance of the vacuum, and ‘z’ is the impedance of the transmission line.

The velocity factor is the ratio of the speed of waves in transmission line to the speed of light in the vacuum.

The electromagnetic wave in a transmission line may reflect at the end. If the load is smaller than the line impedance, a reflection will occur with a 180-degree phase shift.

The forwarding wave and reflection of the incoming wave travels back and forth along the wire, which can form a standing wave.

The charge density wave, which involves alternating compression and expansions, similar to a sound wave in the air, is a kind of longitudinal wave.

The charge density wave is a kind of sound wave. The oscillation is in and opposite to the direction of the wave propagation.

The electromagnetic field wave is a kind of transverse wave. The oscillation of its electric field and magnetic field is perpendicular to the direction of propagation of the wave.

Chapter

# 22

## The electron impedance

The electric charge density wave and magnetic charge density wave are kinds of sound waves. The wave speed has an inverse relationship with the characteristic of electromagnetic impedance.

The characteristic electromagnetic impedance of a vacuum is a constant:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{22.1}$$

What is the characteristic impedance of electron? From the equation (20.7), in Quantum Hall effect, the characteristic impedance of two dimensional electron gases is the ratio of the magnetic charge and electric charge.

Let us make an assumption that the electron characteristic impedance is the ratio of the magnetic charge and electric charge along with the spin direction. Let us now calculate the ratio of magnetic charge and electric charge inside the electron as follows:

Based on the equation (2.7), we have the electron's electric charge in the sphere of radius 'r':

$$Q_e(r,\pi) = -e[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{22.2}$$

The electric charge has the same distribution in both the north pole and south pole. In the north pole area of sphere of radius 'r,' the electric charge is:

$$Q_e(r,0 \to \frac{\pi}{2}) = -\frac{1}{2}e[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{22.3}$$

In the south pole area of the electron sphere of radius 'r', the electric charge is:

$$Q_e(r,\frac{\pi}{2} \to \pi) = -\frac{1}{2}e[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{22.4}$$

Based on the equation (3.7), the magnetic charge distribution in a sphere of radius ‘r’ is zero:

$$Q_m(r, 0 \to \pi) = 0 \tag{22.5}$$

In the north pole area of sphere of radius ‘r,’ the magnetic charge is:

$$Q_m(r, \frac{\pi}{2}) = -g[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{22.6}$$

The south pole area of sphere of radius ‘r,’ the magnetic charge is:

$$Q_m(r, \frac{\pi}{2} \to \pi) = g[1-(1+\frac{r}{a_e})\exp(-\frac{r}{a_e})] \tag{22.7}$$

Thus, within the north pole area of a sphere of any radius r, the ratio of magnetic charge and electric charge is:

$$\frac{Q_m(r, 0 \to \frac{\pi}{2})}{Q_e(r, 0 \to \frac{\pi}{2})} = \frac{2g}{e} \tag{22.8}$$

The ratio is the constant, it doesn’t depend on the sphere radius ‘r’.

In the south pole area of sphere of radius r, the ratio of magnetic charge and electric charge is:

$$\frac{Q_m(r, \frac{\pi}{2} \to \pi)}{Q_e(r, \frac{\pi}{2} \to \pi)} = -\frac{2g}{e} \tag{22.9}$$

The ratio is the constant; it doesn’t depend on the sphere radius ‘r’.

So we can see that inside the electron, for a sphere of any radius ‘r’, both the north pole and south pole have the same ratio of magnetic charge and electric charge, but with opposite signs.

Per our assumption that the electromagnetic characteristic impedance inside of an electron is the ratio of magnetic charge and electric charge along with the spin, which is:

$$Z = \frac{2g}{e} \tag{22.10}$$

As we know, ge=h, thus:

$$Z = \frac{2h}{e^2} \tag{22.11}$$

As we know the fine structure constant is:

$$\alpha = \frac{e^2}{2\varepsilon_0 hc} \tag{22.12}$$

Thus:

$$Z = \frac{1}{\alpha\varepsilon_0 c} \tag{22.13}$$

Thus:

$$Z = \frac{1}{\alpha}\sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{22.14}$$

Thus:

$$\frac{Z_0}{Z} = \alpha \tag{22.15}$$

The ratio of vacuum impedance and electron impedance is the alpha fine structure constant.

As we know, the electromagnetic wave velocity varies inversely with the wave impedance, and we also know that the wave velocity in the vacuum is the speed of light, and then we have following relationship:

$$\frac{V_{wave}}{c} = \frac{Z_0}{Z} \tag{22.16}$$

Thus:

$$\frac{V_{wave}}{c} = \alpha \tag{22.17}$$

Thus:

$$V_{wave} = c\alpha \tag{22.18}$$

As we know the velocity factor is the ratio of the speed of waves to the speed of light in a vacuum, we find out that the velocity factor of an electron is the alpha fine structure constant.

As we know, the ratio of magnetic charge unit and electric charge unit is as follows:

$$\frac{g}{e} = \frac{h}{e^2} \qquad (22.19)$$

As we can see, the value $\frac{h}{e^2}$ is also the quantum hall resistance unit, which is also the ratio of the magnetic charge unit and electric charge unit.

Chapter

# 23 The hydrogen atom spectrum

The hydrogen atom electric charge density distribution equation is:

$$\rho_e = \frac{e}{\pi^2}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}\sin\theta \tag{23.1}$$

Hydrogen atom magnetic charge density distribution equation is:

$$\rho_m = \frac{g}{\pi a^2}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}\cos\theta \tag{23.2}$$

The hydrogen atom electromagnetic field equation as follows:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\sin\theta - \hat{\theta}\cos\theta) \tag{23.3}$$

$$\vec{H} = \frac{g}{2\pi\mu_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e}) - \frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\cos\theta + \hat{\theta}\sin\theta) \tag{23.4}$$

As we know, the hydrogen atom spectrum in vacuum has following format:

$$\frac{1}{\lambda_{nm}} = R_H(\frac{1}{m^2} - \frac{1}{n^2}) \tag{23.5}$$

In which $R_H$ is called the Rydberg constant.

The hydrogen atom spectrum is the electromagnetic field wave energy spectrum; the atom spectrum is the result of the electromagnetic field standing wave, which includes both the electric field standing wave and magnetic field standing wave.

According the equation (17.51) and (17.52), we find out that the ratio of the magnetic charge and electric charge within both the north pole and south pole of a hydrogen atom have the same value but opposite sign.

Similar to the electron, let us assume the hydrogen atom's characteristic impedance is the ratio of magnetic charge and electric charge for both north pole and south pole. Thus we can derive the hydrogen atom's electromagnetic characteristic impedance as follows:

$$Z = Z_0 \frac{1}{\alpha} \tag{23.6}$$

Outside the hydrogen atom is the vacuum. The vacuum impedance is:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \;. \tag{23.7}$$

As we know, the velocity has an inverse relationship with impedance, so the wave velocity inside the hydrogen atom has the following relation with the speed of light.

$$\frac{V}{c} = \frac{Z_0}{Z} \tag{23.8}$$

Then we can get:

$$\frac{V}{c} = \alpha \tag{23.9}$$

The inside of the hydrogen atom has constant impedance and wave velocity. Outside the hydrogen atom, exists a vacuum that has light speed as its wave velocity.

Inside of the hydrogen atom, there are both outgoing and incoming waves along the radial direction, which form a standing wave.

Let's introduce a new function which we can temporally call the zeta-exponential function [C] as follows:

$$g_m(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \xi[2(n+m)] \tag{23.10}$$

For m =1 thus:

$$g_1(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \xi[2(n+1)] \tag{23.11}$$

In which

$$\xi[2(n+1)] = \sum_{m=1}^{\infty} \frac{1}{m^{2n+2}} \tag{23.12}$$

The value of $\xi[2(n+1)]$ is always larger than 1, with the increase of the integer n, the value becomes more and more close to the number 1.

As we know, the spherical wave equation [D] is:

$$\nabla^2\psi = \frac{1}{V^2}\frac{\partial^2\psi}{\partial t^2} \tag{23.13}$$

We can see both the following functions are the solution of the spherical wave equation [D]:

$$\frac{1}{r}g_1(i\frac{r-Vt}{a_e}) = \sum_{k=1}^{\infty}\frac{1}{k^2}\frac{1}{r}\exp(i\frac{r-Vt}{k^2 a_e}) \tag{23.14}$$

And

$$\frac{1}{r}g_1(i\frac{r+Vt}{a_e}) = \sum_{k=1}^{\infty}\frac{1}{k^2}\frac{1}{r}\exp(i\frac{r+Vt}{k^2 a_e}) \tag{23.15}$$

The first solution $\frac{1}{r}g_1(i\frac{r-Vt}{a_e})$ is a kind of outgoing wave.

The second solution $\frac{1}{r}g_1(i\frac{r+Vt}{a_e})$ is a kind of incoming wave.

Let us make an assumption that the hydrogen atom electric field with the stationary wave function is as follows:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e})(1+p\operatorname{Re}[g_1(i\frac{r+Vt}{a_e})+g_1(i\frac{r-Vt}{a_e})]-\frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\sin\theta-\hat{\theta}\cos\theta) \tag{23.16}$$

The hydrogen atom magnetic field equation with the stationary wave is as follows:

$$\vec{H} = \frac{g}{2\pi\mu_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e})(1+q\operatorname{Im}[g_1(i\frac{r+Vt}{a_e})-g_1(i\frac{r-Vt}{a_e})])-\frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\cos\theta+\hat{\theta}\sin\theta) \tag{23.17}$$

In which:

$$p = \frac{B}{m_e c^2}\frac{\pi}{2\alpha}\sqrt{1+\frac{4\alpha^2}{\pi^2}} \tag{23.18}$$

And

$$q = \frac{B}{m_e c^2}\sqrt{1+\frac{4\alpha^2}{\pi^2}} \tag{23.19}$$

The B is the positive constant.

Let us define the constant angle $\beta$ as follows:

$$\tan\beta = \frac{\pi}{2\alpha} \tag{23.20}$$

Thus:

$$\sin\beta = \frac{1}{\sqrt{1+\frac{4\alpha^2}{\pi^2}}} \tag{23.21}$$

$$\cos\beta = \frac{2\alpha}{\pi}\frac{1}{\sqrt{1+\frac{4\alpha^2}{\pi^2}}} \tag{23.22}$$

As we know the electron's electric field energy is:

$$U_e = \frac{m_e c^2}{(1+\frac{4\alpha^2}{\pi^2})}\frac{4\alpha^2}{\pi^2} \tag{23.23}$$

Thus:

$$U_e p = B\frac{2\alpha}{\pi}\frac{1}{\sqrt{1+\frac{4\alpha^2}{\pi^2}}} \tag{23.24}$$

Thus:

$$U_e p = B\cos\beta \tag{23.25}$$

As we know, the electron magnetic field energy is:

$$U_m = \frac{m_e c^2}{(1+\frac{4\alpha^2}{\pi^2})} \tag{23.26}$$

Thus:

$$U_m q = B\frac{1}{\sqrt{1+\frac{4\alpha^2}{\pi^2}}} \tag{23.27}$$

$$U_m q = B\sin\beta \tag{23.28}$$

Thus:

$$\vec{E} = \frac{e}{\pi^2\varepsilon_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e})\{1+p\sum_{k=1}^{\infty}\frac{1}{k^2}[\cos(\frac{r+Vt}{k^2a_e})+\cos(\frac{r-Vt}{k^2a_e})]\}-\frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\sin\theta-\hat{\theta}\cos\theta) \tag{23.29}$$

$$\vec{H}=\frac{g}{\pi\mu_0}[\frac{1}{a_e}\exp(-\frac{r}{a_e})\{1+q\sum_{k=1}^{\infty}\frac{1}{k^2}[\sin(\frac{r+Vt}{k^2a_e})-\sin(\frac{r-Vt}{k^2a_e})]\}-\frac{1}{a_P}\exp(-\frac{r}{a_P})]\frac{1}{r}(\hat{r}\cos\theta+\hat{\theta}\sin\theta) \tag{23.30}$$

Thus:

$$\vec{E}=\frac{e}{\pi^2\varepsilon_0}\{\frac{1}{a_e}\exp(-\frac{r}{a_e})[1+p\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\cos(\frac{Vt}{k^2a_e})]-\frac{1}{a_P}\exp(-\frac{r}{a_P})\}\frac{1}{r}(\hat{r}\sin\theta-\hat{\theta}\cos\theta)) \tag{23.31}$$

$$\vec{E}=\frac{e}{\pi^2\varepsilon_0}\{\frac{1}{a_e}\exp(-\frac{r}{a_e})-\frac{1}{a_P}\exp(-\frac{r}{a_P})+p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\cos(\frac{Vt}{k^2a_e})\}\frac{1}{r}(\hat{r}\sin\theta-\hat{\theta}\cos\theta)) \tag{23.32}$$

$$\vec{H}=\frac{g}{\pi\mu_0}\{\frac{1}{a_e}\exp(-\frac{r}{a_e})-\frac{1}{a_P}\exp(-\frac{r}{a_P})+q\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}[\cos(\frac{r}{k^2a_e})\sin(\frac{Vt}{k^2a_e})]\}\frac{1}{r}(\hat{r}\cos\theta+\hat{\theta}\sin\theta) \tag{23.33}$$

Let us define the function f(r) as:

$$f(r)=\frac{1}{a_e}\exp(-\frac{r}{a_e})-\frac{1}{a_P}\exp(-\frac{r}{a_P}) \tag{23.34}$$

Thus:

$$\vec{E}=\frac{e}{\pi^2\varepsilon_0}\{f(r)+p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\cos(\frac{Vt}{k^2a_e})\}\frac{1}{r}(\hat{r}\sin\theta-\hat{\theta}\cos\theta)) \tag{23.35}$$

$$\vec{H}=\frac{g}{\pi\mu_0}\{f(r)+q\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\sin(\frac{Vt}{k^2a_e})\}\frac{1}{r}(\hat{r}\cos\theta+\hat{\theta}\sin\theta) \tag{23.36}$$

The free electron electric field total energy is:

$$U_e=\frac{e^2}{\pi^3\varepsilon_0a_e} \tag{23.37}$$

As we know, the electric field energy density is:

$$u_e=\frac{1}{2}\varepsilon_0E^2 \tag{23.38}$$

Thus:

$$u_e=\frac{1}{2}\frac{e^2}{\pi^4\varepsilon_0}\frac{1}{r^2}\{\frac{1}{a_e}\exp(-\frac{r}{a_e})-\frac{1}{a_P}\exp(-\frac{r}{a_P})+p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\cos(\frac{Vt}{k^2a_e})\}^2 \tag{23.39}$$

$$u_e=\frac{1}{2}\frac{e^2}{\pi^4\varepsilon_0}\frac{1}{r^2}\{f(r)+p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2a_e})\cos(\frac{Vt}{k^2a_e})\}^2 \tag{23.40}$$

$$u_e = \frac{1}{2}\frac{e^2}{\pi^4 \varepsilon_0}\frac{1}{r^2}\{f(r)^2 + 2f(r)p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e}) +$$

$$+ p^2\frac{1}{a_e^2}\exp(-\frac{2r}{a_e})[\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e})]^2\} \quad (23.41)$$

$$u_e = \frac{1}{2}\frac{e^2}{\pi^4 \varepsilon_0}\frac{1}{r^2}\{f(r)^2 + 2f(r)p\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e}) +$$

$$+4p^2\frac{1}{a_e^2}\exp(-\frac{2r}{a_e})[\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\cos(\frac{Vt}{m^2 a_e})\cos(\frac{Vt}{n^2 a_e})]^2\} \quad (23.42)$$

$$u_e = \frac{1}{2\pi r^2}\{f(r)^2 a_e U_e + 2pf(r)U_e\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e}) +$$

$$+4\frac{B^2}{m_e c^2}\frac{1}{a_e}\exp(-\frac{2r}{a_e})[\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\cos(\frac{Vt}{m^2 a_e})\cos(\frac{Vt}{n^2 a_e})]^2\} \quad (23.43)$$

$$u_e = \frac{1}{2\pi r^2}\{f(r)^2 a_e U_e + 2Bf(r)\cos\beta\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e}) +$$

$$+4\frac{B^2}{m_e c^2}\frac{1}{a_e}\exp(-\frac{2r}{a_e})[\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\cos(\frac{Vt}{m^2 a_e})\cos(\frac{Vt}{n^2 a_e})]^2\} \quad (23.44)$$

The free electron magnetic field energy is:

$$U_m = \frac{g^2}{\pi \mu_0 a_e} \quad (23.45)$$

As we know, the magnetic field energy density is:

$$u_m = \frac{1}{2}\mu_0 H^2 \quad (23.46)$$

Thus:

$$u_m = \frac{1}{2}\frac{g^2}{\pi^2 \mu_0}\frac{1}{r^2}\{f(r) + q\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\sin(\frac{Vt}{k^2 a_e})\}^2 \quad (23.47)$$

$$u_m = \frac{1}{2}\frac{g^2}{\pi^2 \mu_0}\frac{1}{r^2}\{[f(r)]^2 + 2qf(r)\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\sin(\frac{Vt}{k^2 a_e}) +$$

$$+ q^2\frac{1}{a_e^2}\exp(-\frac{2r}{a_e})[\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\sin(\frac{Vt}{k^2 a_e})]^2\} \quad (23.48)$$

$$u_m = \frac{1}{2}\frac{g^2}{\pi^2 \mu_0}\frac{1}{r^2}\{[f(r)]^2 + 2qf(r)\frac{1}{a_e}\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\sin(\frac{Vt}{k^2 a_e}) +$$

$$+4q^2\frac{1}{a_e^2}\exp(-\frac{2r}{a_e})\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\sin(\frac{Vt}{m^2 a_e})\sin(\frac{Vt}{n^2 a_e})\} \quad (23.49)$$

$$u_m = \frac{1}{2\pi r^2}\{[f(r)]^2 a_e U_m + 2f(r)B\sin\beta\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\sin(\frac{Vt}{k^2 a_e}) +$$

$$+4\frac{B^2}{m_e c^2}\frac{1}{a_e}\exp(-\frac{2r}{a_e})\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\sin(\frac{Vt}{m^2 a_e})\sin(\frac{Vt}{n^2 a_e})\} \quad (23.50)$$

As we know the electromagnetic field energy density is as follows:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \quad (23.51)$$

Thus:

$$u = \frac{1}{2\pi r^2}\{f(r)^2 a_e m_e c^2 + 2Bf(r)\exp(-\frac{r}{a_e})\sum_{k=1}^{\infty}\frac{2}{k^2}\cos(\frac{r}{k^2 a_e})\cos(\frac{Vt}{k^2 a_e} - \beta) +$$

$$+4\frac{B^2}{m_e c^2}\frac{1}{a_e}\exp(-\frac{2r}{a_e})\sum_{m,n=1}^{\infty}\frac{1}{m^2 n^2}\cos(\frac{r}{m^2 a_e})\cos(\frac{r}{n^2 a_e})\cos[\frac{Vt}{a_e}(\frac{1}{m^2} - \frac{1}{n^2})]\} \quad (23.52)$$

From here we can see that the hydrogen atom's electromagnetic energy spectrum frequency has the following format:

$$v_{mn} = \frac{V}{a_e}(\frac{1}{m^2} - \frac{1}{n^2}) \quad (23.53)$$

As we already know, the hydrogen atom's wave velocity is as follows:

$$V = c\alpha \quad (23.54)$$

Thus:

$$v_{nm} = \frac{c\alpha}{a_e}(\frac{1}{m^2} - \frac{1}{n^2}) \quad (23.55)$$

For the electromagnetic field wave in a vacuum, the wavelength is as follows:

$$\frac{1}{\lambda_{nm}} = \frac{\alpha}{a_e}(\frac{1}{m^2} - \frac{1}{n^2}) \quad (23.56)$$

As we know from (9.15), we have:

$$a_e = a_0(1+\frac{4\alpha^2}{\pi^2}) \tag{23.57}$$

Thus:

$$\frac{1}{\lambda_{nm}} = \frac{\alpha}{a_0}\frac{1}{(1+\frac{4\alpha^2}{\pi^2})}(\frac{1}{m^2}-\frac{1}{n^2}) \tag{23.58}$$

Compared with the hydrogen atom spectrum formula (23.5), we can get:

$$R_H = \frac{\alpha}{a_e} \tag{23.59}$$

Then:

$$R_H = \frac{\alpha}{a_0(1+\frac{4\alpha^2}{\pi^2})} \tag{23.60}$$

**Reference**

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Chapter

# 24 Time-space and energy-momentum

In our book, we make an assumption that all the particles, electrons, protons and neutrons have electromagnetic origin.

We know all the material was made by the molecules, the molecule was consisted of atoms, the atom consists of electrons, protons and neutrons, and so all material has electromagnetic origin.

Electric charge and magnetic charge are the most basic properties of electromagnetic field.

1. Based on the charge distribution, we can get field distribution.
2. Based on the field distribution, we can get field velocity.
3. The charge moves according to the field velocity.
4. Because of the velocity, the charge distribution keeps changing.
5. Because the charge distribution changes, the field then changes.
6. Because the field changes, then the field velocity changes.

Based on the distribution of electric charge and magnetic charge in space, and according the Gauss Law for electric field and magnetic field, we can get the electric field and magnetic field distribution.

And then, based on the electric field and magnetic field space distributions, we can get the electromagnetic field velocity, and then all of the electric and magnetic charges will move according to the electromagnetic field velocity.

The electric charge and magnetic charge space distribution will keep changing, then the electric field and magnetic field will keep changing, and then electromagnetic field velocity will also keep changing.

However the change happened, the electric charge and magnetic charge will keep conserved.

All the basic particles include electrons, protons and neutrons, they are no longer regarded as point-like particles, but are instead considered to have a spherical electromagnetic field with the continuum distribution of electric charge and magnetic charge.

The key relationship between time-space and energy-momentum is the velocity of the electromagnetic field.

As we know, the electromagnetic field velocity has the following relationship with energy-momentum:

$$\vec{V} = \frac{\vec{p}}{\rho} \tag{24.1}$$

In which ‘ $\vec{p}$ ’ is the electromagnetic field momentum density, and ‘ $\rho$ ’ is the electromagnetic field mass density.

As we know, the electromagnetic field velocity has the following relationship with time-space:

$$\vec{V} = \frac{d\vec{r}}{dt} \tag{24.2}$$

We have the following symmetrical relationship between time-space and energy-momentum:

| Field Velocity $\vec{V}$ | |
|---|---|
| Time T | Energy E |
| Space $\vec{R}$ | Momentum $\vec{P}$ |
| Absolute time S | Rest energy U |

The key between these relations is electromagnetic field velocity.

As we know, the rest energy density ‘ $u_0$ ’ is defined as:

$$u_0^2 = u^2 - c^2 p^2 \tag{24.3}$$

In which the electromagnetic field energy density ‘u’ is:

$$u = \rho c^2 \tag{24.4}$$

In which ‘ $\rho$ ’ is the electromagnetic field mass density

$$u_0 = \rho_0 c^2 \tag{24.5}$$

In which ‘ $\rho_0$ ’ is the electromagnetic field rest mass density, thus:

$$\vec{p} = \frac{1}{c^2} \frac{u_0 \vec{v}}{\sqrt{1 - v^2/c^2}} \tag{24.6}$$

Thus:

$$\vec{p} = \frac{\rho_0 \vec{v}}{\sqrt{1 - v^2/c^2}} \tag{24.7}$$

The symmetrical part of the rest energy is the absolute time 's,' which is defined as:

$$c^2 ds^2 = c^2 dt^2 - dr^2 \tag{24.8}$$

Thus the time 't' and the absolute time 's' have following relationship:

$$dt = \frac{ds}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{24.9}$$

The energy always has positive value, and the time is always one-directional. Time flows from the past into the future along with the direction of field velocity, the arrow of time is the direction of the field velocity.

# Appendix A

**The gamma distribution function:**

$$f(r,k,a)=\frac{1}{a\Gamma(k)}(\frac{r}{a})^{k-1}\exp(-\frac{r}{a})$$

In which k is the shape parameter, a is the scale parameter, $\Gamma$ is the gamma function which has the formula:

$$\Gamma(k)=\int_0^\infty r^{k-1}e^{-r}dr$$

# Appendix B

**The cumulative gamma distribution:**

$$f(r,h,a) = 1 - \exp(-\frac{r}{a})\sum_{k=0}^{h}\frac{1}{k!}(\frac{r}{a})^{k}$$

For example, when h=1, thus:

$$f(r,1,a) = 1 - (1+\frac{r}{a})\exp(-\frac{r}{a})$$

# Appendix C

**Zeta-exponential function**

As we know the exponential function is:

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

The Riemann Zeta function is:

$$\xi(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

Combine the exponential function and zeta function, and then we can have a new function, which we temporally called the zeta-exponential function as follows:

$$g_m(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \xi[2(n+m)]$$

Then we will get:

$$g_m(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{k=1}^{\infty} \frac{1}{k^{2n+2m}}$$

Thus:

$$g_m(x) = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} [\sum_{n=0}^{\infty} \frac{x^n}{n!} \frac{1}{k^{2n}}]$$

Thus:

$$g_m(x) = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} [\sum_{n=0}^{\infty} \frac{1}{n!} (\frac{x}{k^2})^n]$$

Thus:

$$g_m(x) = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \exp(\frac{x}{k^2})$$

Thus, we have:

$$g_m(ix) = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \exp(i \frac{x}{k^2})$$

The above equation is the Fourier serial of the complex zeta-exponential function.

$$\text{Re}[g_m(ix)] = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \cos(\frac{x}{k^2})$$

$$\text{Im}[g_m(ix)] = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \sin(\frac{x}{k^2})$$

As we know:

$$[g_m(x)]' = \sum_{k=1}^{\infty} \frac{1}{k^{2m+2}} \exp(\frac{x}{k^2})$$

Thus:

$$[g_m(x)]' = g_{m+1}(x)$$

As we know for the exponential function, we have:

$$\exp(ix) = \cos x + i \sin x$$

$$\text{Re}[\exp(\text{ix})] = \text{cosx}$$

$$\text{Im}[\exp(\text{ix})] = \text{sinx}$$

Similar to the exponential function, the complex zeta-exponential function has the following relationship:

$$g_m(ix) = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} [\cos(\frac{x}{k^2}) + i \sin(\frac{x}{k^2})]$$

$$g_m(ix) = [\sum_{k=1}^{\infty} \frac{1}{k^{2m}} \cos(\frac{x}{k^2})] + i[\sum_{k=1}^{\infty} \frac{1}{k^{2m}} \sin(\frac{x}{k^2})]$$

$$\text{Re}[g_m(ix)] = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \cos(\frac{x}{k^2})$$

$$\text{Im}[g_m(ix)] = \sum_{k=1}^{\infty} \frac{1}{k^{2m}} \sin(\frac{x}{k^2})$$

for m =1 and $x = \dfrac{r - vt}{a}$, thus we have

$$g_1(i\frac{r - vt}{a}) = [\sum_{k=1}^{\infty} \frac{1}{k^2} \cos(\frac{r - vt}{k^2 a})] + i[\sum_{k=1}^{\infty} \frac{1}{k^2} \sin(\frac{r - vt}{k^2 a})]$$

for m =1 and $x = \frac{r+vt}{a}$, thus we have

$$g_1(i\frac{r+vt}{a}) = [\sum_{k=1}^{\infty}\frac{1}{k^2}\cos(\frac{r+vt}{k^2 a})] + i[\sum_{k=1}^{\infty}\frac{1}{k^2}\sin(\frac{r+vt}{k^2 a})]$$

# Appendix D

**Spherical wave equation:**

In the spherical polar coordinate axes ($r, \theta, \varphi$), the gradient $\nabla$ is:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial \phi}$$

The spherical wave equation is:

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Where '$v$' is the speed of the wave which propagate through radial direction, for the special case in which it has angular symmetry (not dependent on the $\theta, \phi$), then we will have the following wave equation:

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Thus:

$$\frac{\partial^2 (r\psi)}{\partial t^2} = v^2 \frac{\partial^2 (r\psi)}{\partial r^2}$$

Thus, we have the following general solution:

$$\psi = \frac{A}{r}\exp(i\frac{r - vt}{a}) + \frac{B}{r}\exp(i\frac{r + vt}{a})$$

Thus:

$$\psi = \frac{A}{r}\exp[i(kr - wt)] + \frac{B}{r}\exp[i(kr + wt)]$$

In which

$$k = \frac{1}{a}$$

Where k is the wave number.

$$w = \frac{v}{a}$$

Where w is the angular frequency.

# Appendix E

The gradient $\nabla$ in spherical coordinate is:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial \phi}$$

In which the spherical polar coordinate axes is ($r$, $\theta$, $\phi$).

# Index

## T



## V



## W



## Z

## What is the electron spin?

The electron has both intrinsic electric field and intrinsic magnetic field. The electron's intrinsic electromagnetic field has both energy and angular momentum.

The electron spin is the electron's electromagnetic field angular momentum.

The electron's self-energy is the electron's electromagnetic field energy.

The electron has electromagnetic origin.

The electron is the smallest magnet, oriented in the same direction that the electron is spinning. The electron's south pole has one unit positive magnetic charge 'g', the north pole has one unit negative magnetic charge '–g'.

The multiple of electric charge unit 'e' and magnetic charge unit 'g' equals Planck's constant 'h'.

Both the electron's electric charge and magnetic charge have the continuum spherical distribution inside the electron.

The electron as whole has one unit negative electric charge 'e'.

The electron as whole has zero magnetic charge.

All the particles that make up physical materials include electrons, protons and neutrons have electromagnetic origin. Thus all materials have electromagnetic origin.